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## INVESTIGATION OF RADIATIVE INTERACTION IN LAMINAR FLOWS USING MONTE CARLO SIMULATION

By

Jiwen Liu and S. N. Tiwari

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Old Dominion University  
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## **PREFACE**

This research work was conducted in cooperation with the Fluid Mechanics Division of the NASA Langley Research Center during the period January 1991 to December 1991. Certain changes in basic formulations were included during 1992 and essential findings were reported at the 28th National Heat Conference in San Diego, California, in August 1992. Additional results were obtained during the spring of 1993.

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# **INVESTIGATION OF RADIATIVE INTERACTION IN LAMINAR FLOWS USING MONTE CARLO SIMULATION**

**By**

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## **ABSTRACT**

The Monte Carlo method(MCM) is employed to study the radiative interactions in fully developed laminar flow between two parallel plates. Taking advantage of the characteristics of easy mathematical treatment of the MCM, a general numerical procedure is developed for nongray radiative interaction. The nongray model is based on the statistical narrow band model with an exponential-tailed inverse intensity distribution. To validate the Monte Carlo simulation for nongray radiation problems, the results of radiative dissipation from the MCM are compared with two available solutions for a given temperature profile between two plates. After this validation, the MCM is employed to solve the present physical problem and results for the bulk temperature are compared with available solutions. In general, good agreement is noted and reasons for some discrepancies in certain ranges of parameters are explained.

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## NOMENCLATURE

### Latin Symbols

$e_{b\omega}$	Planck function, $(\text{W}\cdot\text{cm}^2)/\text{cm}^{-1}$
$\bar{h}$	equivalent heat transfer coefficient, $\text{W}/\text{cm}^2\cdot\text{K}$
$k$	line intensity to spacing ratio; also thermal conductivity, $\text{erg}/\text{cm}\cdot\text{sec}\cdot\text{K}$
$P$	gas pressure, atm
$q_R$	radiative flux, $\text{W}/\text{cm}^2$
$q_w$	wall heat flux, $\text{erg}/(\text{cm}^2\cdot\text{s})$
$R$	random number
$S$	integrated band intensity, $\text{atm}^{-1}\cdot\text{cm}^{-2}$
$T$	temperature, K
$T_w$	wall temperature, K
$T_b$	bulk temperature, K
$u$	streamwise velocity, $\text{cm}/\text{sec}$
$v$	transverse velocity, $\text{cm}/\text{sec}$
$x$	flow direction
$y$	transverse direction

### Greek symbols

$\alpha$	thermal diffusivity, $\text{cm}^2/\text{sec}$
$\beta$	line width to spacing ratio
$\gamma$	half-width of an absorption line, $\text{cm}^{-1}$
$\delta$	equivalent line spacing, $\text{cm}^{-1}$
$\epsilon$	emissivity
$\theta$	dimensionless temperature, $(T-T_w)/(q_w L/k)$
$\theta_b$	dimensionless bulk temperature
$\kappa$	absorption coefficient, $\text{cm}^{-1}$
$\kappa_p$	Planck mean absorption coefficient, $\text{cm}^{-1}$
$\xi$	dimensionless coordinate, $y/L$

$\rho$	reflectivity; also density, g/cm <sup>3</sup>
$\omega$	wavenumber, cm <sup>-1</sup>

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## 1. INTRODUCTION

There is a renewed interest in investigating various aspects of radiative energy transfer in participating media. Radiative interactions become important in many engineering problems involving high temperature gases. Recent interest lies in the areas of design of high pressure combustion chambers and high enthalpy nozzles, entry and reentry phenomena, hypersonic propulsion, and defence oriented research.

Analyses and solutions of practical problems involving radiative interactions with other modes of heat transport require efficient and versatile numerical techniques. Many available methods are based on procedures developed for the analysis of radiative transport. These include P-N method, discrete ordinate method, zoning method, finite element method, and Monte Carlo method (MCM). A review of the state of available solution techniques is given by Howell (1988).

The MCM is a probabilistic method which can exactly simulate all important physical processes. In this method, the mathematical treatment of numerical analysis is easy and the difficulty for radiative transfer problems in complex geometries can be circumvented easily. It is due to these advantages that the MCM has been applied to solve many radiative transfer problems. The earliest application of this method on radiative transfer problems was made by Howell and Perlmutter (1964a). Radiative problems of increasing complexity which have been investigated by this method have appeared in the literature (Perlmutter and Howell, 1964; Howell and Perlmutter, 1964b; Steward and Cannon, 1971; Dunn, 1983; Gupta et al., 1983; Taniguchi et al., 1991). Studies on reducing the computational time by using this method are also available (Kobiyama et al., 1979; Kobiyama et al., 1986). The gray gas assumption, however, is made in most radiative transfer analyses. In many practical applications, this approximation is too crude to provide quantitative predictions. This is because, in most cases, the gas radiation properties strongly depend on the wavenumber.

The application of the MCM for analysis of radiative transfer in real gases has received little attention. Howell and Perlmutter (1964) are the first to take into account the effect of

nongray radiation. The spectral absorption coefficient of hydrogen at very high pressure and temperature was obtained by experimental procedures and used for analysis of radiation by the MCM. Another technique which approximates a real gas by the weighted-sum-of-gray-gases approach was also modeled using the MCM (Steward and Cannon, 1971). Recently, Taniguchi et al. (1991) have applied a simplified form of the Elsasser band model to investigate the problem of radiative equilibrium in a plane-parallel system. It is pointed out that the temperature profile in the gas layer can be predicted accurately by the MCM.

This work is motivated by our interest to apply a general and accurate nongray model to investigate radiative interactions using the MCM. Consequently, a statistical narrow band model with an exponential-tailed-inverse intensity distribution is applied to calculate gas radiative properties in the present study. Consideration of the major infrared bands and evaluation of line parameters of these bands simulate the true nature of participating species and transfer processes. These properties are used to solve radiative transfer problems by the MCM. As an example of the application of the statistical narrow band model with the Monte Carlo method, radiative interactions in fully developed laminar flow between two parallel plates are studied. The reason for choosing this case is the availability of approximate solutions in the literature (Tiwari, 1985; Tiwari et al., 1990). Before this application, however, the radiative dissipation solution obtained from the MCM is compared with that from other methods in order to establish the validity of the MCM for nongray radiative analyses.

## 2. THEORETICAL FORMULATION

The physical problem considered is the steady-state energy transfer in laminar, incompressible, constant properties, fully developed flow of absorbing-emitting gases between two parallel plates (Fig. 2.1). The condition of uniform surface heat flux is assumed such that the surface temperature varies in the axial direction. The energy equation for this case can be expressed as (Cess and Tiwari, 1972; Sparrow and Cess, 1978)

$$\rho C_p \left( u \frac{\partial T}{\partial x} + v \frac{\partial T}{\partial y} \right) = k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_R}{\partial y} \quad (2.1)$$

where  $u$  and  $v$  denote the  $x$  and  $y$  components of velocity, respectively.

For a fully-developed flow,  $v=0$ , and  $u$  is given by the well-known parabolic profile as

$$u = 6u_m(\xi - \xi^2); \quad \xi = y/L \quad (2.2)$$

where  $u_m$  represents the mean fluid velocity. Also, for the flow of a perfect gas with uniform heat flux,  $\partial T/\partial x$  is constant and is given by

$$\partial T/\partial x = (2\alpha q_w)/(u_m L k) \quad (2.3)$$

A combination of Eqs. (2.1)-(2.3), therefore, results in

$$k \frac{\partial^2 T}{\partial y^2} - \frac{\partial q_R}{\partial y} - \frac{12q_w}{L}(\xi - \xi^2) = 0 \quad (2.4)$$

Equation (2.4) is the governing energy equation for the parallel plates geometry. The boundary conditions for this problem can be expressed as

$$T(0) = T(L) = T_w; \quad \frac{\partial T}{\partial y}(y = L/2) = 0 \quad (2.5)$$

It should be noted that all boundary conditions given in Eq. (2.5) are not independent, any two convenient conditions can be used to obtain specific solutions.

The radiative transfer term in the energy equation makes computation difficult because it turns the differential equation into an integro-differential equation. One exception is for the case

of a gray medium. In this case, the equation for radiative transfer is expressed as (Sparrow and Cess, 1978)

$$\frac{1}{\kappa_p^2} \frac{\partial^2 q_R(y)}{\partial y^2} - \frac{9}{4} q_R(y) = \frac{3}{2} \sigma \frac{\partial T^4}{\partial y} \quad (2.6)$$

where  $\kappa_p$  is the Planck mean absorption coefficient. For black walls and  $T_{w1}=T_{w2}$ , the boundary conditions for Eq. (2.6) become

$$q_R(L/2) = 0; \quad \frac{3}{2} q_R(0) = \frac{1}{\kappa_p} (\partial q_R / \partial y)_{y=0} \quad (2.7)$$

In the present study, attention is directed to apply the MCM to solve the radiative transfer term for nongray as well as gray media. Before going into detailed numerical analysis to solve the energy equation as well as the radiative transfer term, it is essential to define the quantity of primary interest and choose the appropriate radiative transfer models.

For incompressible flow problems, the quantity of primary interest is the bulk temperature of the gas. For a fully-developed flow between parallel plates, this is expressed as

$$\theta_b = (T_b - T_w) / (q_w L / k) = 6 \int_0^1 \theta(\xi) (\xi - \xi^2) d\xi \quad (2.8)$$

where  $q_w = \bar{h}(T_w - T_b)$ , and  $\bar{h}$  represents the equivalent heat transfer coefficient ( $\text{W}/\text{cm}^2\text{—K}$ ).

The study of radiative transmission in nonhomogeneous gaseous systems requires a detailed knowledge of the absorption, emission and scattering characteristics of the specific gas. Several models are available in the literature to represent the absorption emission characteristics of molecular species. The gray gas model is probably the simplest model to employ in radiative transfer analyses. For a nonuniform temperature field, the mean absorption coefficient used for the optically thin radiation is the modified Planck mean absorption coefficient which for black bounding surfaces is defined as (Tien, 1968; Sparrow and Cess, 1978; Tiwari, 1992)

$$\kappa_m(T, T_w) = \kappa_p(T_w)(T_w/T) \quad (2.9)$$

where  $\kappa_p(T)$  represents the Planck mean absorption coefficient. For a multiband system of a homogeneous gas,  $\kappa_p(T)$  is expressed as

$$\kappa_p(T) = P \sum_{i=1}^n [e_b(\omega_i, T) S_i(T)] / (\sigma T^4) \quad (2.10)$$

where  $n$  represents the number of vibration-rotation bands,  $e_b(\omega_i, T)$  is the Planck function evaluated at the  $i$ th band center, and  $S_i(T)$  is the integrated band intensity of the  $i$ th band. Equation (2.10) is modified to apply to a mixture of different gases as

$$\kappa_p(T) = \sum_j P_j \left\{ \sum_{i=1}^n [e_b(\omega_i, T) S_i(T)] \right\}_j / (\sigma T^4) \quad (2.11)$$

where  $j$  denotes the number of species in the mixture and  $P_j$  is the partial pressure of  $j$ th species.

In many practical applications, the radiative transfer by hot molecular gases such as  $H_2O$  and  $CO_2$  involves vibration-rotation bands that are difficult to model by a gray gas model due to the strong wavenumber dependent properties of the bands. In such cases, an appropriate nongray gas model needs to be invoked.

The nongray gas models include line-by-line models, narrow band models, and wide band models. The solution of line-by-line formulation requires considerably large computational resources. The wide band model and band absorptance correlations also present some disadvantages (Zhang et al., 1988). Various wide and narrow band models have been tested with line-by-line calculations (Soufiani et al., 1985; Soufiani and Taine, 1987). Accurate results for temperature and heat flux distribution are obtained with the narrow band model which assumes the absorption lines to be randomly placed and the intensities to obey an exponential-tailed-inverse distribution. The transmissivity of a homogeneous and isothermal column of length  $l$  due to gas species  $j$ , averaged over  $[\omega - (\Delta\omega/2), \omega + (\Delta\omega/2)]$ , is then given by

$$\bar{\tau}_\omega^j = \exp \left[ -\frac{\bar{\beta}}{\pi} \left( \sqrt{\left( 1 + \frac{2\pi x_j P l \bar{k}}{\bar{\beta}} \right)} - 1 \right) \right] \quad (2.12)$$

where  $x_j$  represents the mole fraction of the absorbing species  $j$  and  $P$  is total pressure;  $\bar{k}$  and  $\bar{\beta} = 2\pi\bar{\gamma}/\bar{\delta}$  are the band model parameters which take into account the spectral structure of the

gas. Parameters  $\bar{k}$  and  $1/\bar{\delta}$  generated from a line-by-line calculation have been published for  $H_2O$  and  $CO_2$  (Ludwig et al., 1973; Hartmann et al., 1984; Soufiani et al., 1985). The mean half-width  $\bar{\gamma}$  is given by (Soufiani et al., 1985)

$$\bar{\gamma}_{H_2O} = 0.066 \frac{P}{P_s} \left\{ 7.0 x_{H_2O} \frac{T_s}{T} + [1.2(x_{H_2O} + x_{N_2}) + 0.8x_{O_2} + 1.6x_{CO_2}] \sqrt{\left(\frac{T_s}{T}\right)} \right\} \quad (2.13)$$

and

$$\bar{\gamma}_{CO_2} = \frac{P}{P_s} \left(\frac{T_s}{T}\right)^{0.7} [0.07x_{CO_2} + 0.058(x_{N_2} + x_{O_2}) + 0.15x_{H_2O} + 0.15x_{H_2O}] \quad (2.14)$$

where  $P_s$  and  $T_s$  designate standard pressure and temperature (1 atm, 296 K).

For a nonisothermal and inhomogeneous column, the Curtis-Godson approximation leads to accurate results if pressure gradients are not too large. Basically, this approach consists of transformation of such a column into an equivalent isothermal and homogeneous one. Effective band model parameters  $\bar{k}_e$  and  $\bar{\beta}_e$  are introduced by averaging  $\bar{k}$  and  $\bar{\beta}$  over the optical path  $U$  of the column as

$$U(l) = \int_0^l P(y) x_i(y) dy \quad (2.15)$$

$$\bar{k}_e = \frac{1}{U(l)} \int_0^l P(y) x_i(y) \bar{k}(y) dy \quad (2.16)$$

$$\bar{\beta}_e = \frac{1}{\bar{k}_e U(l)} \int_0^l P(y) x_i(y) \bar{k}(y) \bar{\beta}(y) dy \quad (2.17)$$

The transmissivity of this equivalent column is then calculated from Eq. (2.12).

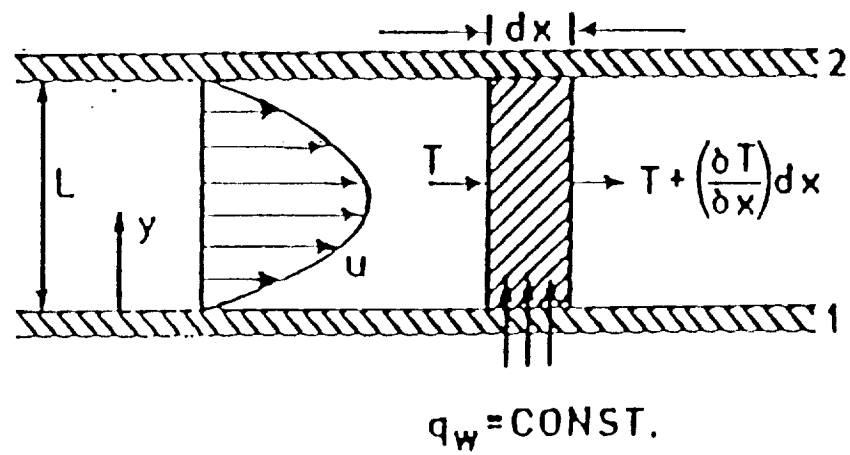


Fig. 2.1 Laminar flow between parallel plates with constant wall heat flux.

### 3. NUMERICAL ANALYSIS

There are two levels to the numerical method proposed here. The first is concerned with the finite difference discretization and solution of the energy equation, while the second is due to the numerical evaluation of the radiative flux term that is included in the energy equation.

The energy equation, Eq. (2.4), is discretized by a finite volume technique. The domain between two parallel plates is divided equally into  $N$  finite volume elements. For the  $i$ th finite volume element  $\delta V_i$ , the energy equation can be written as

$$k \frac{T_{i+1} - 2T_i + T_{i-1}}{\Delta y} - \frac{12q_w \Delta y}{L} (\xi_i - \xi_i^2) + Q_{V-\delta V_i} + Q_{A-\delta V_i} - Q_{\delta V_i} = 0 \quad (3.1)$$

where the conductive heat transfer is discretized by the central difference scheme and the radiative heat transfer consists of  $Q_{V-\delta V_i}$ ,  $Q_{A-\delta V_i}$  and  $Q_{\delta V_i}$  terms. The quantity  $Q_{V-\delta V_i}$  is the total radiant energy absorbed in  $\delta V_i$  which was emitted by all volume elements in the domain including  $\delta V_i$  itself,  $Q_{A-\delta V_i}$  is the total radiant energy absorbed in  $\delta V_i$  which was emitted by all surfaces, and  $Q_{\delta V_i}$  is the radiant energy emitted by  $\delta V_i$ . Since the temperatures of two parallel plates are given, the energy balances result in a set of simultaneous equations equal to the total number of finite volume elements. Each equation contains the unknown quantity temperature which cannot be calculated by independent means, so an iterative solution is necessary. Before going to the analysis of energy solution, it is essential to evaluate the radiative energy interchange in each equation.

The typical method for handling radiative interchange between elements of volumes and/or surfaces is to evaluate the multiple integrals which describe the interchange by some type of numerical integration technique. This, usually, is a good approach for simple problems. An alternative method is used here. Radiative emission in the domain is simulated using the MCM to obtain  $Q_{V-\delta V_i}$  and  $Q_{A-\delta V_i}$  directly.

The MCM uses bundles of energy to simulate the actual physical processes of radiant emission and absorption of energy occurring in the domain. The energy per bundle is simply



some fraction of the total or net radiant energy emitted throughout the domain per unit time. The history of an energy bundle from its emission until it is finally absorbed is determined by a series of random numbers which are generated every time a decision with respect to position, direction, wavenumber, path length, reflection or absorption is required. The relationships between the random number and position, direction, wavenumber, path length, reflection or absorption have been developed by Howell (1968) and Siegel and Howell (1981). For gray media, the spectral dependence of the energy bundle is neglected, so the simulation processes are little simpler. But when the effect of spectral properties is taken into account, the difficulty of the simulation processes increases.

First, we need to determine the radiant energy  $Q_{A_i}$  of surface  $A_i$  which is supposed to be absorbed by the volume elements and the radiant energy  $Q_{\delta V_i}$  emitted by the finite volume element  $\delta V_i$ . For the narrow band model, the absorption bands of the gas are divided into spectral ranges  $\Delta\omega$  wide; each is centered at  $\omega^k$  and characterized by the superscript  $k$ ;  $\Delta\omega$  is chosen in such a manner that the Planck function  $e_{b\omega}(T)$  is constant in the range  $[\omega - (\Delta\omega/2), \omega + (\Delta\omega/2)]$ . Then,  $Q_{A_i}$  is given as

$$Q_{A_i} = \sum_k \epsilon_i e_{b\omega^k}(T_i) \Delta\omega \cdot A_i \quad (3.2)$$

where  $\epsilon_i$  is the emissivity of surface  $A_i$ . The radiant energy  $Q_{\delta V_i}$  in the  $i$ th finite volume element is expressed as

$$Q_{\delta V_i} = 4 \sum_k e_{b\omega^k}(T_i) \left[ \int_0^1 (1 - \bar{\tau}_{\omega^k}) \mu d\mu \right] \Delta\omega \cdot \delta V_i \quad (3.3)$$

where  $\mu$  is the cosine of the angle between the  $y$  axis and the direction of a column. The quantity  $\bar{\tau}_{\omega^k}$  is the mean spectral transmissivity of a column of length  $\Delta y/\mu$  in the  $k$ th band.

When an energy bundle is emitted from the  $i$ th volume element, its wavenumber is determined from

$$R_\omega = \frac{4 \int_0^\omega e_{b\omega}(T_i) \left[ \int_0^1 (1 - \tau_\omega) \mu d\mu \right] d\omega \cdot \delta V_i}{Q_{\delta V_i}} \quad (3.4)$$

where  $R_\omega$  is a random number. The transmissivity  $\bar{\tau}_{\omega k}$  in Eq. (2.12) is the value averaged over the spectral range  $\Delta\omega$  and is different from spectral transmissivity  $\tau_\omega$ . But,  $\tau_\omega$  can be taken to be equal to the  $\bar{\tau}_{\omega k}$ , and this is treated as spectral transmissivity at the center of band if  $\omega$  is in the region of the  $k$ th band. Equation (3.4) is solved for  $\omega$  each time an  $R_\omega$  is chosen. The computing time becomes too large for practical calculations since both  $\tau_\omega$  and  $e_{b\omega}$  are very complex functions of  $\omega$  and the number of bundles usually is very large. To circumvent this problem, interpolation and approximation methods are employed. We first choose different values of  $\omega$  and obtain the corresponding values of  $R_\omega$  from Eq. (3.4). Then, a smooth curve is constructed to match these data points, and  $\omega$  values are easily obtained from this curve for selected values of  $R_\omega$ . This work is accomplished by using cubic spline interpolation CSDEC in IMSL Library Package (Anonymous, 1987). Similar calculation procedures are applied to determine wavenumber when an energy bundle is emitted from a surface.

For variable properties along the optical path, the distance  $X$  the energy bundle travels before absorption in the gas is given by (Howell, 1968; Siegel and Howell, 1981)

$$\ln R_l = -\frac{\bar{\beta}'}{\pi} \left( \sqrt{\left( 1 + \frac{2\pi x_i P X \bar{k}'}{\bar{\beta}'} \right)} - 1 \right) \quad (3.5)$$

where  $\bar{k}'$  and  $\bar{\beta}'$  are the mean Curtis-Godson parameters averaged over the distance  $X$ . When solving this equation numerically,  $X$  values are assumed and  $\bar{k}'$  and  $\bar{\beta}'$  are calculated for different  $X$  values until the right side of the equation is equal to the left side. But if radiative properties are constant along the optical path, the  $X$  values can be evaluated directly by solving Eq. (3.5).

The method for determination of the direction, reflection or absorption of an energy bundle for nongray radiation problems is similar to gray radiation problems for isotropic media and diffusive surface; the details of the procedure are given by Howell (1968) and Siegel and Howell (1981).

The temperature distribution in the domain is assumed before Monte Carlo simulation of radiative interchange begins. The total number of energy bundles in each finite volume or surface element is proportional to its radiant energy. Every energy bundle is followed according to the above formulations until absorption occurs. A large number of bundles is considered to

satisfactorily represent the radiation emitted by a volume or surface element. The total number of energy bundles absorbed by each element multiplied by the energy per bundle gives the interchange of radiation among the volume and /or surface elements. For the  $i$ th element, the total number of energy bundles absorbed multiplied by the energy per bundle gives  $Q_{V-\delta V_i}$  and  $Q_{A-\delta V_i}$ .

From the Monte Carlo simulation,  $Q_{V-\delta V_i}$  and  $Q_{A-\delta V_i}$  are obtained based on the assumed temperature distribution; a new temperature distribution, which is included in convective and conductive heat transfer as well as in  $Q_{\delta V_i}$ , is obtained by solving a set of non-linear equations. This work is accomplished by using the NEQNF routine which solves a system of non-linear equations in IMSL Library Package (Anonymous, 1987). The change in local temperature in each iteration of the calculation is determined and when the maximum change is less than  $10^{-4}$ , the solution is considered to have converged.

#### 4. RESULTS AND DISCUSSIONS

Based on the theoretical and numerical analysis described in the previous sections, two computer codes were developed to solve nongray radiation problems. The calculation was carried out on a Sun Workstation. In order to validate the MCM for nongray participating media, a code was written first to evaluate the radiation dissipation term,  $-\partial q_R/\partial y$ , for the case of water vapor between two parallel plates with constant emissivities. The same physical problem has been considered previously and different solutions have been obtained (Zhang et al., 1988; Kim et al., 1991) with the narrow band model used in this study. The Monte Carlo solutions are obtained and compared with the available solutions. This is essential for code verification before the MCM is employed to solve the problems involving radiative interactions with other modes of heat transfer. Usually the only difference for most numerical methods for radiation problems is in the evaluation of the term  $-\partial q_R/\partial y$ .

A nearly parabolic temperature profile ( Fig. 4.1) used by Zhang et al. (1988) and Kim et al. (1991) is also used here to calculate the radiation dissipation term,  $-\partial q_R/\partial y$ . The plate spacing is divided into 20 uniform finite volume elements for the numerical calculation. The total number of energy bundles is 50,000 and the CPU time is on the order of 1000s for each physical problem. Figure 4.2 shows the comparison of the Monte Carlo solutions and results of Zhang et al. (1988). The plates spacing is assumed to be 0.05 m and the pressure of water vapor is 1.0 atm. The reflectivities of two plates are assumed to be equal, and two cases with different reflectivities are taken into account. For each case, it is found that in the regions near the plates, the Monte Carlo solution is little lower than the correlated and non-correlated results of Zhang et al. (1988). The correlated results are the results in which the correlations between the radiating gas property and the intensity are taken into account. In the central region of the plates, the Monte Carlo results are essentially between the correlated and non-correlated results.

Figure 4.3 shows the comparison of the Monte Carlo solutions and S-N discrete ordinates solutions by Kim et al. (1991) for the case with equal reflectivities of the two plates ( $\rho_1 =$

$\rho_2 = 0.5$ ). The plate spacing and vapor pressure are kept at 0.5 m and 1.0 atm, respectively. The Monte Carlo solution agrees well with the S-N discrete ordinates solution in the region near the plate. In the central region, a little difference between the two solutions is noted, with the Monte Carlo solution being slightly higher. A unsymmetrical case with different plate reflectivities  $\rho_1 = 0.9$ ,  $\rho_2 = 0.0$  was also considered (see Tiwari and Liu, 1992). A very good agreement between the two solutions was noted in all regions. These solutions are not shown in Fig. 4.3 because of clarity. From the comparisons presented in Figs. 4.2 and 4.3, it is concluded that the Monte Carlo solutions agree very well with other solutions and the method can be employed to solve the nongray radiation problems accurately.

After the validation of the MCM, a second code was developed to consider the gray and nongray radiation problem combined with conductive and convective heat transfer. For the case of black walls, gray analytical solutions and nongray approximate solutions based on the method of variation of parameters are available in the literature (Tiwari, 1985 and 1992; Tiwari et al., 1990). In this study, the Monte Carlo solutions are compared with these results for identical conditions. The absorbing—emitting media considered are pure  $H_2O$  and  $CO_2$ . The results are expressed in terms of the non-dimensional bulk temperature. The plate spacings considered range from 0.01 cm to 100.0 cm. The domain is divided into 40 finite volume elements with equal thicknesses. The total number of energy bundles selected is 50,000 for nongray and 200,000 for gray simulation. The amount of energy per bundle depends on the temperature. One of the important parameters related to the temperature distribution is the heat flux from the plates; care should be taken to choose this heat flux. In the solutions of Tiwari (1985 and 1992) and Tiwari et al. (1990), the assumption of linearized radiation was made and the radiative properties were considered to be independent of temperature. In order to facilitate the comparison between the Monte Carlo solution and the approximate solution, different values of heat flux at the wall are chosen when the plate spacings change. The CPU time requirement for a converged solution with a specific plate spacing is on the order of ten seconds for the gray case if the differential emissive power emission method (DPE method-Kobiyama et al., 1979; Kobiyama et al., 1986)

is applied and on the order 1000 seconds for the nongray case. The numerical experiments conducted in this study indicate that the DPE method can reduce the CPU time about an order of magnitude compared to the normal method without loss in the accuracy of results.

Figures 4.4 and 4.5 show comparisons between the gray analytical solutions, nongray approximate solutions, and the corresponding Monte Carlo solutions for different temperatures and pressures. The medium considered is  $\text{CO}_2$ . In Fig. 4.4, the pressure of  $\text{CO}_2$  is kept at 1.0 atm but plate temperature changes as 500 and 1000 K for the gray and nongray cases. In Fig. 4.5, the wall temperature is kept at 1000 K but the pressure changes as 1.0 and 5.0 atm. The figures show that the predictions by the MCM are very close to the analytical solutions for gray cases at different temperatures and pressures. For nongray cases, the Monte Carlo solutions compare favorably with the approximate solutions. However, some differences are also noted. In the intermediate optical regions, the predictions by the MCM are a slightly higher. Figures 4.6 and 4.7 show the results for  $\text{HO}_2$ . The physical conditions are the same as for Figs. 4.4 and 4.5, respectively. Similar to the results of  $\text{CO}_2$ , excellent agreements are found for the gray cases. For the nongray cases, the agreement between different solutions appears better in the optically thin region than other optical regions. The predictions by the MCM are usually higher than those by the approximate solutions.

Form the comparative results presented in Figs. 4.4–4.7, a correct trend is seen for the Monte Carlo solution and a good agreement is noted between the predictions of the MCM and other methods for the two gases at different temperatures and pressures. Obviously, the agreement between solutions for nongray medium is not as good as that for the gray medium. There are several reasons that contribute to this trend. First, in the approximate solutions, the exponential kernel approximation was employed in the radiative flux equation, and the energy equation was solved by the method of variation of parameters which is also an approximate method. But, the MCM exactly simulates the radiative interchange process and the solution of the energy equation is also accurate. Second, in the approximate solution, the exponential wide band model was employed, the spectral discretization was too wide and this leads to errors in the

radiative flux distribution for some optical lengths. But in the MCM, the relationship between the spectral properties and wavenumber is considered in an exact manner. Third, the exponential wide band model is based on the narrow band model. If the total band absorptance is evaluated based on numerical quadrature from the narrow band model and compared with that obtained by the exponential wide band model, the difference is quite obvious. As for as the MCM itself is concerned, a statistical error may still exist, although attempts are made to select statistical quantities properly to deliberately reduce it.

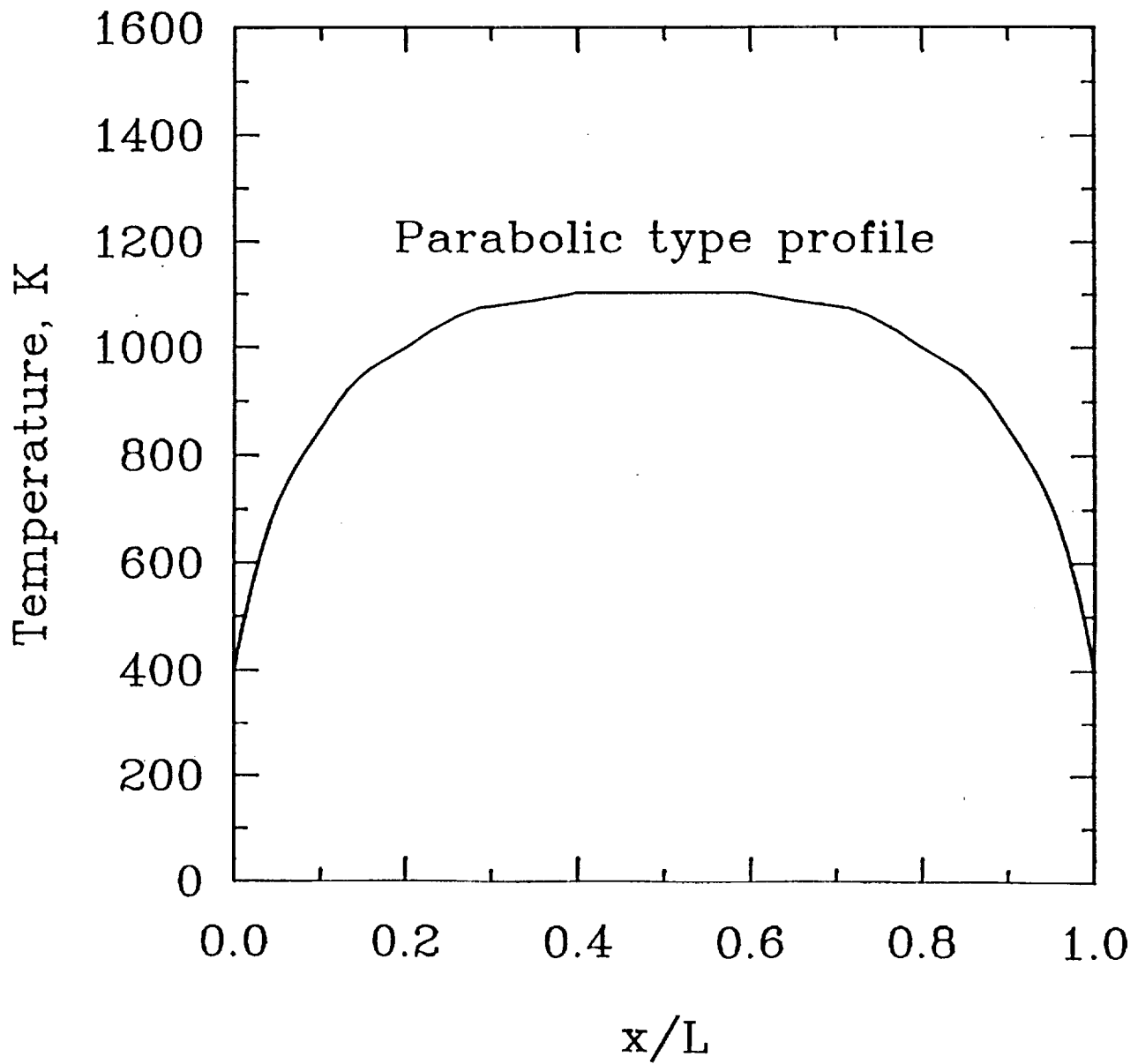


Fig. 4.1 Temperature profile.



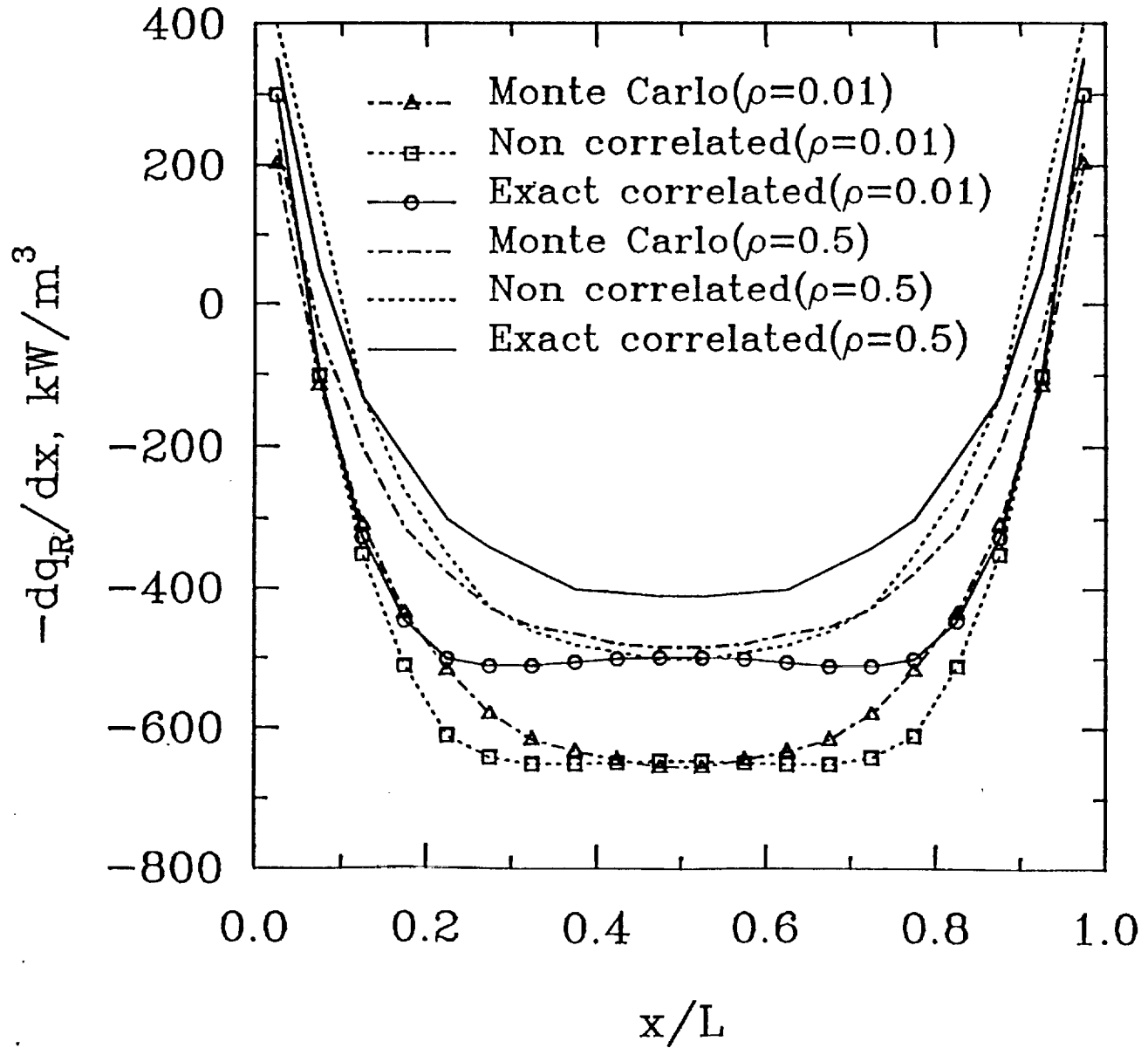


Fig. 4.2 Comparison of radiative dissipation in pure  $\text{H}_2\text{O}$  for  $L=0.05\text{m}$ ,  $p=p_{\text{H}_2\text{O}}=1.0\text{atm}$ .

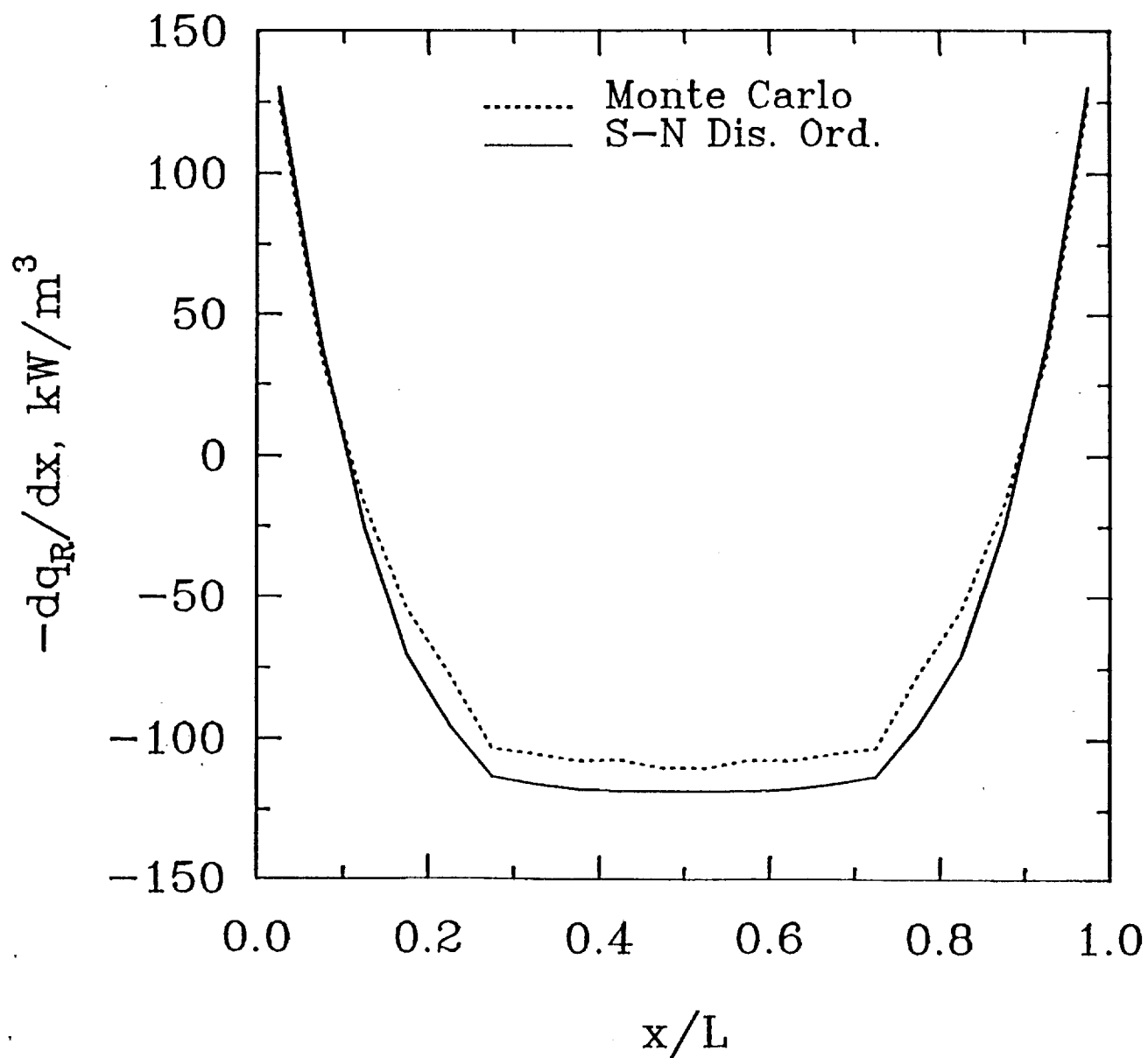


Fig. 4.3 Comparison of radiative dissipation in pure  $\text{H}_2\text{O}$  for  $L=0.5\text{m}$ ,  $p=p_{\text{H}_2\text{O}}=1.0\text{atm}$ .

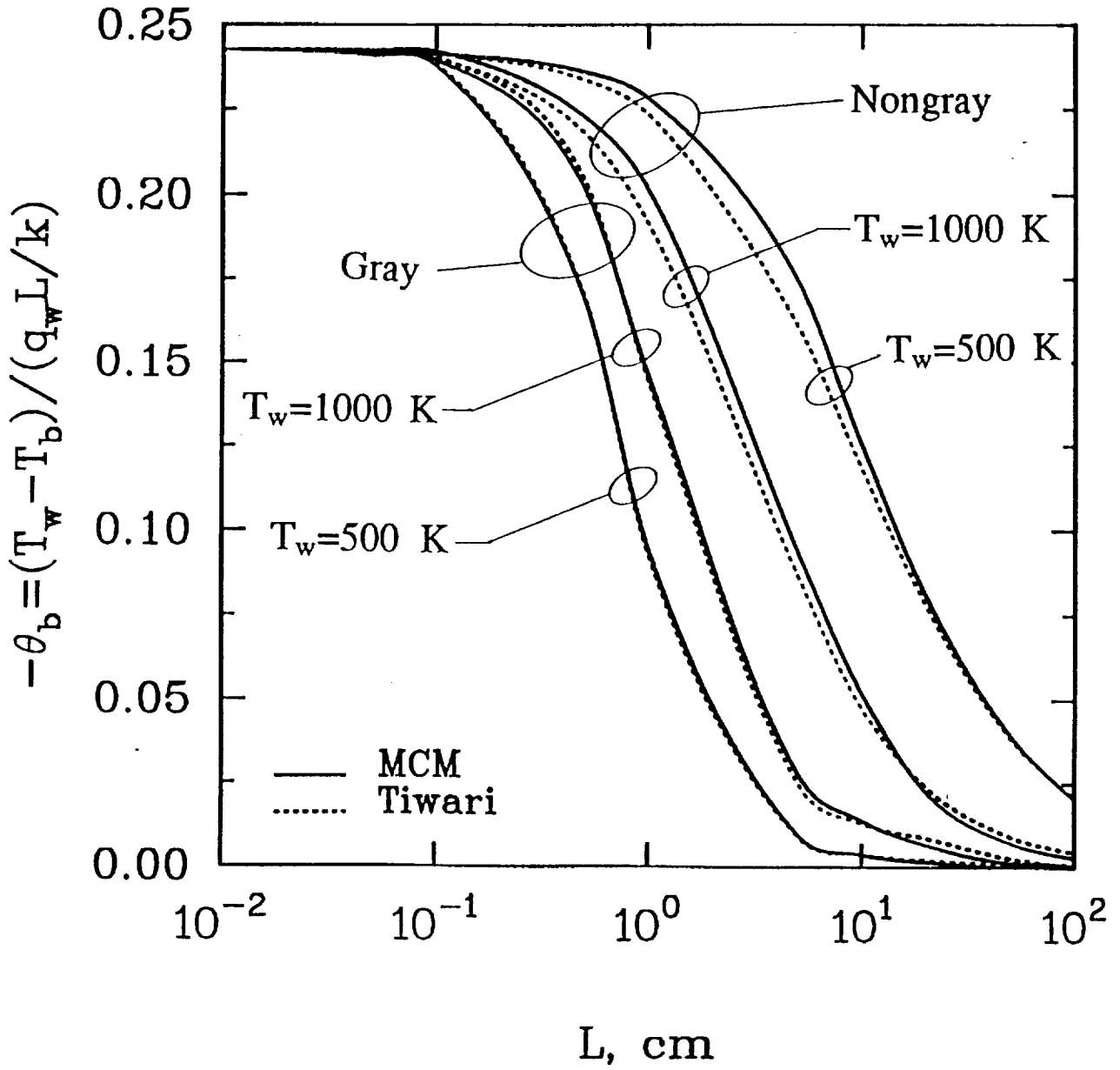


Fig. 4.4 Comparison of gray and nongray solutions for  $\text{CO}_2$  at  $P=1 \text{ atm}$ .

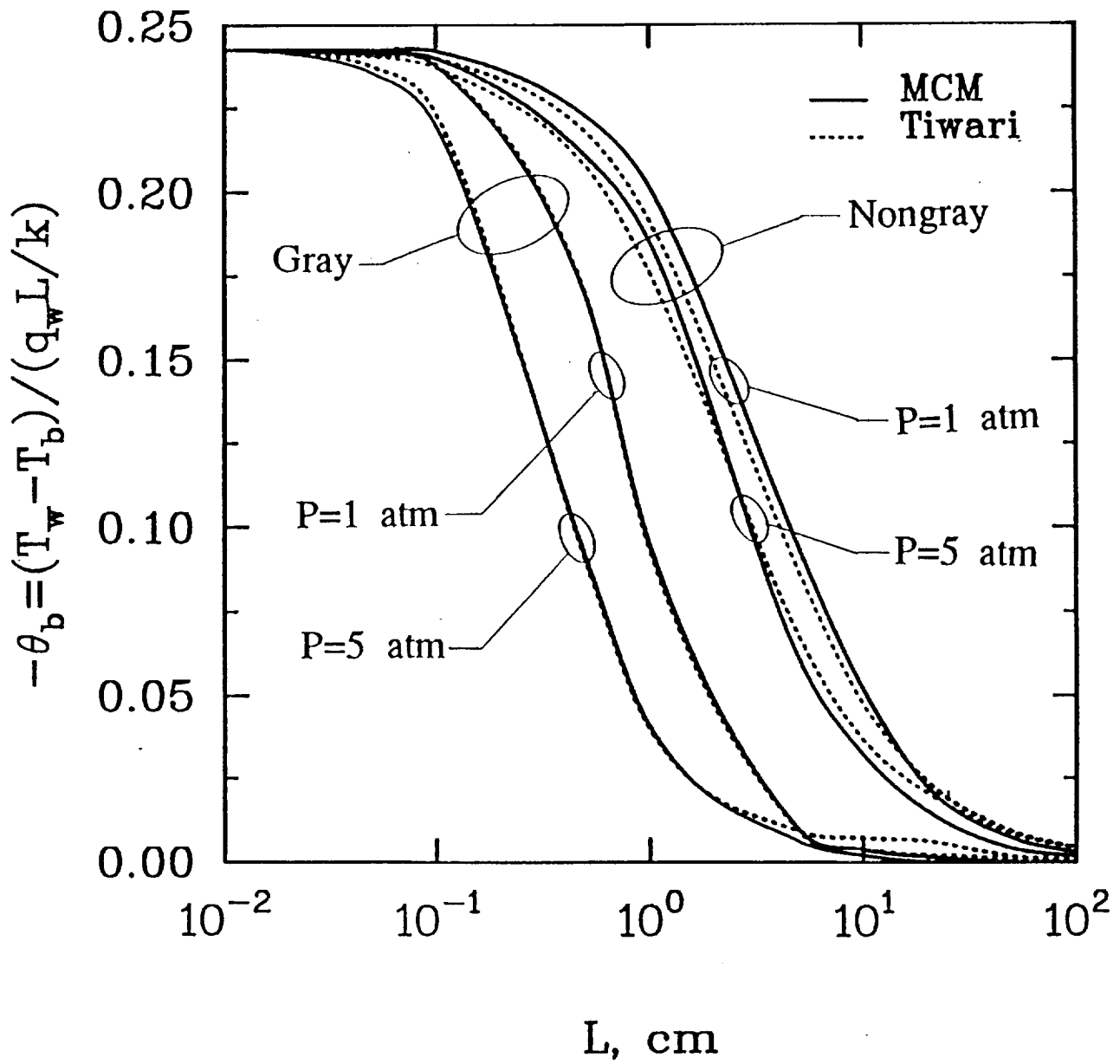


Fig. 4.5 Comparison of gray and nongray solutions for  $\text{CO}_2$  at  $T_w=1000$  K.

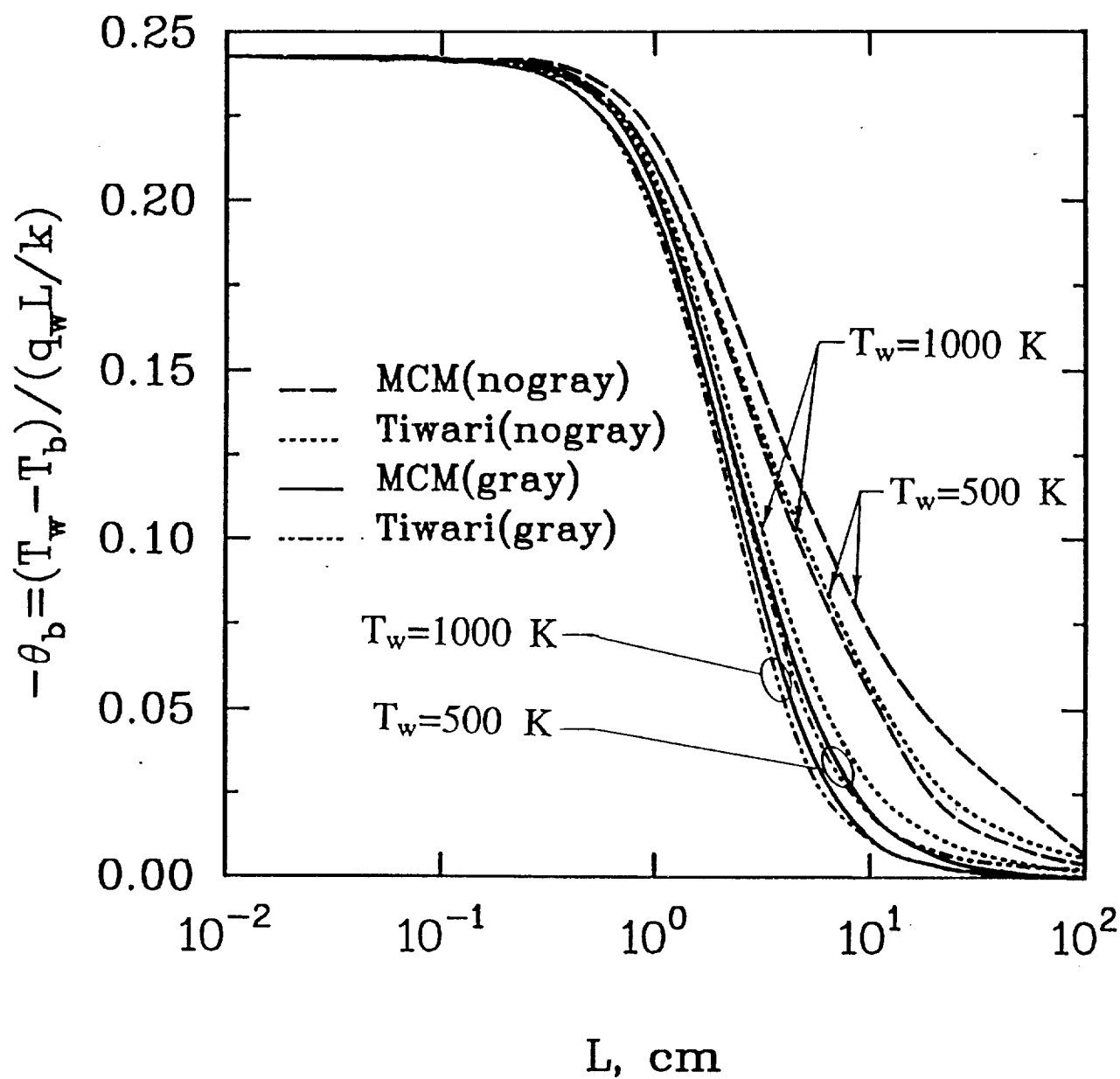


Fig. 4.6 Comparison of gray and nongray solutions for  $\text{H}_2\text{O}$  at  $P=1 \text{ atm}$ .

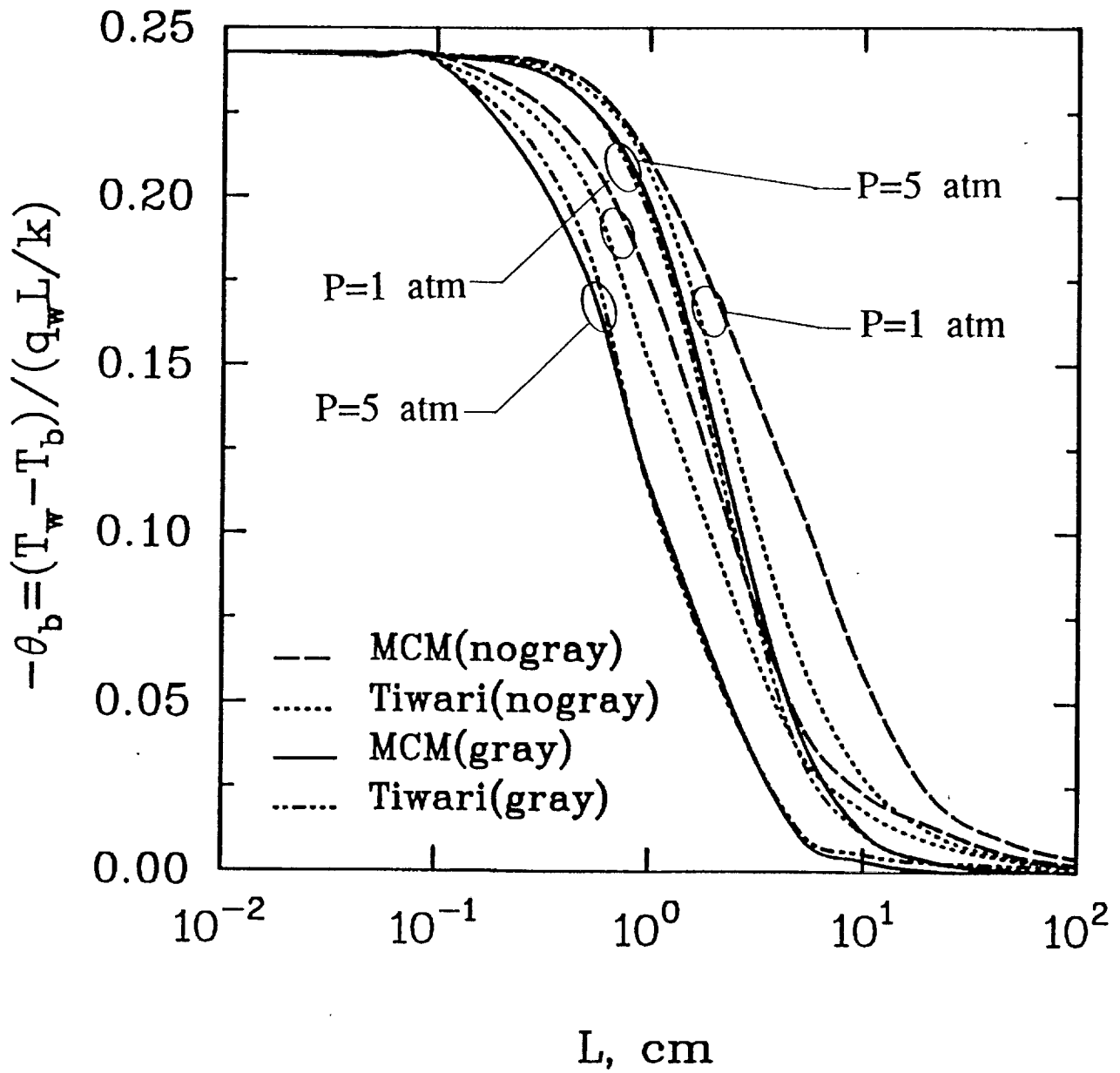


Fig. 4.7 Comparison of gray and nongray solutions for  $H_2O$  at  $T_w=1000$  K.

## 5. CONCLUDING REMARKS

The MCM is employed to study radiative interactions in steady fully developed laminar flow of absorbing-emitting species between two parallel plates. Gray as well as nongray models for radiation absorption are considered. The Monte Carlo solutions for gray medium are found to be in excellent agreement with other solutions. The nongray formulation is based on a statistical narrow band model with an exponential-tailed-inverse intensity distribution. The nongray results using this model have been obtained for the first time in conjunction with the MCM. In general, the results are in good agreement with nongray results of other studies. Some possible reasons for differences between various solutions are provided.

## REFERENCES

- Anonymous, 1987, IMSL MATH/LIBRARY Reference Manual, IMSL, Inc., Houston, TX.
- Cess, R. D., and Tiwari, S. N., 1972, "Infrared Radiative Energy Transfer in Gases," Advances in Heat Transfer, Vol. 8, Academic Press, New York.
- Dunn, W. L., 1983, "Inverse Monte Carlo Solutions for Radiative Transfer in Inhomogeneous Media," Journal of Quantitative Spectroscopy and Radiative Transfer, Vol. 29, No. 1, January 1983, pp. 19-26.
- Gupta, R. P., Wall, T. F., and Truelove, J. S., 1983, "Radiative Scatter by Fly Ash in Pulverized-Coal-Fired Furnaces: Application of the Monte Carlo Method to Anisotropic Scatter," International Journal of Heat and Mass Transfer, Vol. 26, No. 11, November 1983, pp. 1649-1660.
- Hartmann, J. M., Levi Di Leon, R., and Taine, J., 1984, "Line-by-line and Narrow-band Statistical Model Calculations for  $H_2O$ ", Journal of Quantitative Spectroscopy and Radiative Transfer, Vol. 32, No. 2, February 1984, pp. 119-127.
- Howell, J. R., 1968, "Application of Monte Carlo to Heat Transfer Problems," Advances in Heat Transfer, Vol. 5, Academic Press, New York.
- Howell, J. R., 1988, "Thermal Radiation in Participating Media: The Past, the Present, and Some Possible Futures," Journal of Heat Transfer, Vol. 110, November 1988, pp. 1220-1229.
- Howell, J. R., and Perlmutter, M., 1964a, "Monte Carlo Solution of Thermal Transfer through Radiant Media between Gray Walls," Journal of Heat Transfer, Vol. 86, February 1964, pp. 116-122.
- Howell, J. R., and Perlmutter, M., 1964b, "Monte Carlo Solution of Radiant Heat Transfer in a Nongray Nonisothermal Gas with Temperature Dependent Properties," American Institute of Chemical Engineers Journal, Vol. 10, No. 4, July 1964, pp. 562-567.



Kim, T. K., Menart, J. A., and Lee, H. S., 1991, "S-N Discrete Ordinate Solutions of Nongray Radiative Transfer with Diffusely Reflecting Walls", Proceedings of the 28th National Heat Transfer Conference, Minneapolis, Minnesota, ASME HTD-Vol. 16, July 1991, pp. 79-87.

Kobiyama, M., Taniguchi, H., and Saito, T., 1979, "The Numerical Analysis of Heat Transfer Combined with Radiation and Convection", Bulletin of the Japan Society of Mechanical Engineers, Vol. 22, No. 167, May 1979, pp. 707-714.

Kobiyama, M., 1986, "A Study on the Reduction of Computing Time of the Monte Carlo Method Applied to the Radiative Heat Transfer", Bulletin of the Japan Society of Mechanical Engineers, Vol. 29, No. 255, September 1986, pp. 3000-3006.

Ludwig, C. B., Malkmus, W., Reardon, J. E., and Thompson, J. A. L., 1973, "Handbook of Infrared Radiation from Combustion Gases". NASA SP-3080.

Perlmutter, M. and Howell, J. R., 1964, "Radiative Transfer through a Gray Gas between Concentric Cylinders Using Monte Carlo," Journal of Heat Transfer, Vol. 86, May 1964, pp. 169-179.

Siegel, R., and Howell, J. R., 1981, Thermal Radiation Heat Transfer, McGraw-Hill Book Co., New York, 1971; Second Edition, 1981.

Soufiani, A., Hartmann, J. M., and Taine, J., 1985, "Validity of Band-model Calculation for CO<sub>2</sub> and H<sub>2</sub>O Applied to Radiative Properties and Conductive-radiative Transfer", Journal of Quantitative Spectroscopy and Radiative Transfer, Vol. 33, No. 3, March 1985, pp. 243-257.

Soufiani, A., and Taine, J., 1987, "Application of Statistical Narrow-band Model to Coupled Radiation and Convection at High Temperature", International Journal of Heat and Mass Transfer, Vol. 30, No. 3, March 1987, pp. 437-447.

Sparrow, E. M., and Cess, R. D., 1978, Radiation Heat Transfer, Augmented Edition, Hemisphere Publishing Corporation, Washington, D. C..

Steward, F. R., and Cannon, P., 1971, "The Calculation of Radiative Heat Flux in a

Cylindrical Furnace Using the Monte Carlo Method," International Journal of Heat and Mass Transfer, Vol. 14, No. 2, February 1971 , pp. 245–262.

Taniguchi, H., Kudo, K., Otaka, M., Sumarsono, M., and Obata, M., 1991, "Non-gray Analysis on Radiative Energy Transfer through Real Gas Layer by Monte Carlo Method", Proceedings of the 7th International Conference on Numerical Methods of Thermal Problems.

Tien, C. L., 1968, "Thermal Radiation Properties of Gases", Advances in Heat Transfer, Vol. 5, Academic Press, New York.

Tiwari, S. N., 1985, "Radiative Interaction in Transient Energy Transfer in Gaseous System", NASA-CR-176644 NAS 1.26:176644, December 1985.

Tiwari, S. N., 1992, "Radiative Energy Transfer in Molecular Gases," Institute for Computational and Applied Mechanics (ICAM), Old Dominion University, Norfolk, Virginia, ODU/ICAM Report 92–101, February 1992.

Tiwari, S. N. and J. Liu, 1992, "Investigation of Radiative Interaction in Laminar Flows of Nongray Gases Using Monte Carlo Simulation", The 28th ASME/AIChE National Heat Transfer Conference, San Diego, California, August, 1992.

Tiwari, S. N., Singh, D. J. , and Trivedi, P. A., 1990, "Radiative Interactions in Laminar Incompressible and Compressible Internal Flows", AIAA Paper 90–0134, January 1990.

Zhang, L., Soufiani, A., and Taine, J., 1988, "Spectral Correlated and Non-correlated Radiative Transfer in a Finite Axisymmetric System Containing an Absorbing and Emitting Real Gas-particle Mixture", International Journal of Heat and Mass Transfer, Vol. 31, No. 11, November 1988, pp. 2261–2272.